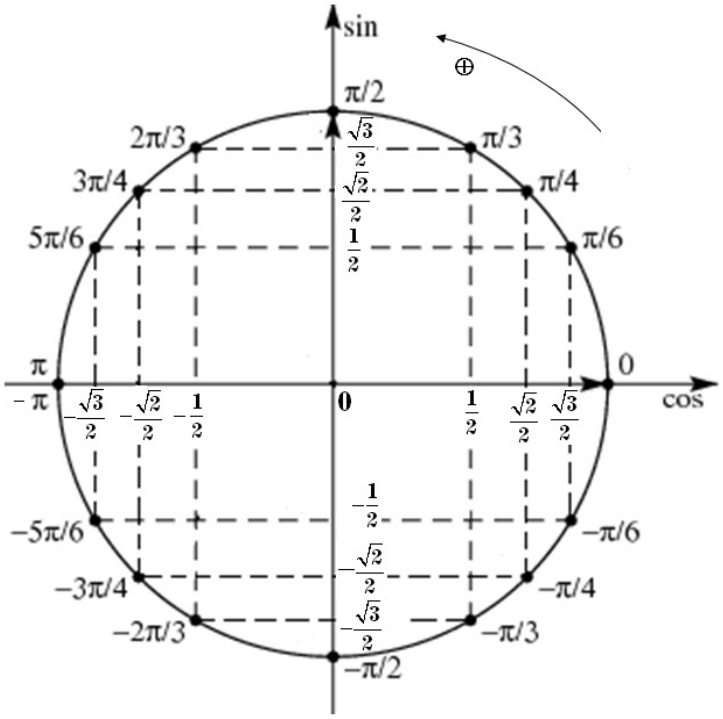
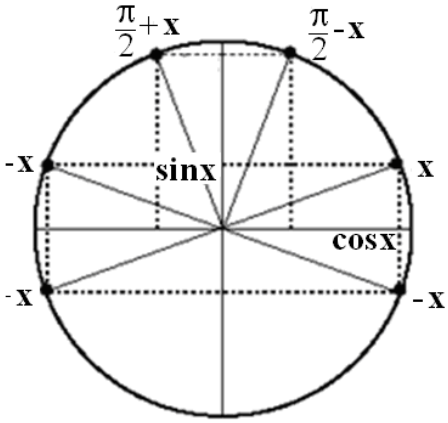
الحساب المثلثي (تذكير)

جدول القيم الاعتيادية و الدائرة المثلثية:



x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

العلاقات بين النسب المثلثية:



	$-x$	$\pi - x$	$\pi + x$	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$
sin	$-\sin x$	$\sin x$	$-\sin x$	$\cos x$	$\cos x$
cos	$\cos x$	$-\cos x$	$-\cos x$	$\sin x$	$-\sin x$

$\cos(x + 2k\pi) = \cos x$ $\sin(x + 2k\pi) = \sin x$ $\tan(x + k\pi) = \tan x$	$\tan x = \frac{\sin x}{\cos x}$ $1 + \tan^2 x = \frac{1}{\cos^2 x}$	$-1 \le \cos x \le 1$ $-1 \le \sin x \le 1$ $\cos^2 x + \sin^2 x = 1$

معادلات مثلثية أساسية:

$\cos x = \cos a \Leftrightarrow x = a + 2k\pi$ أو $x = -a + 2k\pi$

$\sin x = \sin a \Leftrightarrow x = a + 2k\pi$ أو $x = (\pi - a) + 2k\pi$

$\tan x = \tan a \Leftrightarrow x = a + k\pi \quad (k \in \mathbb{Z})$

صيغ تحويل مجموع:

$$\cos(a+b) = \cos a \times \cos b - \sin a \times \sin b$$
$$\sin(a+b) = \sin a \times \cos b + \cos a \times \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$$

$$\cos(a-b) = \cos a \times \cos b + \sin a \times \sin b$$
$$\sin(a-b) = \sin a \times \cos b - \cos a \times \sin b$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \times \tan b}$$

نتائج:

$$\cos 2a = \cos^2 a - \sin^2 a$$
$$= 2\cos^2 a - 1$$
$$= 1 - 2\sin^2 a$$
$$\sin 2a = 2\sin a \times \cos a$$
$$\tan 2a = \frac{2\tan a}{1 - \tan^2 a}$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$
$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

بوضع: $t = \tan\frac{a}{2}$

$$\sin a = \frac{2t}{1+t^2}$$
$$\cos a = \frac{1-t^2}{1+t^2}$$
$$\tan a = \frac{2t}{1-t^2}$$

تحويل جداء إلى مجموع :

$$\cos a \times \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$
$$\sin a \times \sin b = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$
$$\sin a \times \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$
$$\cos a \times \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$$

تحويل مجموع إلى جداء :

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$
$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\sin p - \sin q = 2\cos\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

تحويل: $a\cos x + b\sin x$ $\quad (a,b) \neq (0,0)$

$$a\cos x + b\sin x = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos x + \frac{b}{\sqrt{a^2+b^2}}\sin x\right)$$
$$= \sqrt{a^2+b^2}\cos(x-\alpha)$$

حيث α عدد حقيقي يحقق:

$$\sin\alpha = \frac{b}{\sqrt{a^2+b^2}} \quad \text{و} \quad \cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$$