تمارين مختلفة: التعداد والاحتمالات

1

تمرين 1

$A = \{1, 2, 3\}$ و B مجموعتان منتهيتان

$\text{Card}(A \times B) = \text{Card}(A) \times \text{Card}(B)$ إذن $\text{Card}(A \times B) = 6$ و $\text{Card}(A) = 2$

$\text{Card}(B) = 3$ و $\text{Card}(A \times B) = \text{Card}(A) \times \text{Card}(B)$

حل التمرين

نطرح في

$\text{Card}(A \cap B) = \text{Card}A + \text{Card}B - \text{Card}(A \cup B)$

$\text{Card}(A \cap B) = \text{Card}A + \text{Card}B - \text{Card}(A \times B)$

وبما أن $\text{Card}(A \cup B) = \text{Card}(A \times B)$

فإن $\text{Card}(A \cap B) = \text{Card}A + \text{Card}B - \text{Card}(A \times B)$

وبالتالي $\text{Card}(A \cap B) = 2 + 3 - 6$

فإن $\text{Card}(A \cap B) = 1$

فعدد العناصر المشتركة بين A و B هو واحد.

2

تمرين 2

A_n^k هي عدد الأعداد الطبيعية المكونة من k أرقام مختلفة من n أرقام

$A_n^k = \frac{n!}{(n-k)!}$ و $C_n^k = \frac{A_n^k}{k!}$

حل التمرين

1) نحسب A_7^3 و C_7^3 ثم A_7^4 و C_7^4 ثم A_7^5 و C_7^5

الشروط $n \in \mathbb{N}$ و $k \in \mathbb{N}$ و $n \geq k$

$A_7^3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \times 6 \times 5 = 210$

$C_7^3 = \frac{A_7^3}{3!} = \frac{210}{6} = 35$

$A_7^4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$

$C_7^4 = \frac{A_7^4}{4!} = \frac{840}{24} = 35$

$A_7^5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = 7 \times 6 \times 5 \times 4 \times 3 = 2520$

$C_7^5 = \frac{A_7^5}{5!} = \frac{2520}{120} = 21$

2) نحل في $\mathbb{N}$ المعادلة $C_7^k = 7$

الشروط $n \in \mathbb{N}$ و $k \in \mathbb{N}$ و $n \geq k$

$C_7^k = 7 \Leftrightarrow \frac{A_7^k}{k!} = 7 \Leftrightarrow \frac{\frac{7!}{(7-k)!}}{k!} = 7$

$\Leftrightarrow \frac{7!}{(7-k)!k!} = 7 \Leftrightarrow \frac{7!}{(7-k)!k!} = 7 \times 1$

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